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Newton's *Experimentum Crucis* Reconsidered

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Certain terminological inconsistencies in the teaching of optical theory at the elementary level are traced back to Newton's *Opticks* and shown to derive from an uncritical application of the terminology of the old, established Euclidean geometrical optics to experiments which are primarily concerned with the establishment of a physical theory of light. The "ray" of Euclidean geometrical optics should be considered as a geometrical operator, the working rules of which are laid down in a set of axioms. If the Euclidean ray concept is adapted to those dispersion experiments put forward by Newton in favor of his hypotheses, some general class properties of dispersion phenomena are revealed. From the insight in those class properties, certain counter experiments can be easily recognized which demonstrate that it makes no sense to connect a certain color of the ray with its degree of refrangibility. There is a brief discussion of the significance of such experiments for insight into the part played by pure observation in scientific theory making.

In the public image of the development of physical sciences, Newton's *Opticks* (1704)[1,2] plays a distinguished part, representing an ideal case of experimental verification of hypothetical assumptions. Nevertheless, a closer examination of the hypothetical–deductive structure of the *Opticks* reveals a terminological ambiguity which has greatly influenced later textbooks. As a consequence, a general class property of dispersion phenomena has been overlooked.

This terminological ambiguity also lies behind the obvious contradictions in occasional commentaries by Newton himself to the so called *Experimentum Crucis* (*Experimentum VI*):

> . . . y^{e} designe of it is to show that rays of divers colours do at equall incidences suffer unequall refractions without being split, rarefied, or any ways dilated[3]
>
> . . . you think I brought it, to prove that rays of different colours are differently refrangible: whereas I bring it to prove (wthout respect to colours) y^{t} light consists in rays differently refrangible[4]

In the *Opticks* itself a number of postulates which are logically independent of another, are not properly distinguished. For instance:

1. "White" light is "heterogeneous," i.e., it is a mixture of "spectral lights."
2. Spectral lights are "homogeneous."
3. Spectral lights are specifically refrangible.
4. Spectral lights are specifically colored (in the sense that they give rise to specific color sensations in the eye).

In what follows we shall be concerned with the interpretation of the introductory experiments of the *Opticks* which are usually referred to by the textbooks:

Experimentum III: A beam of light from a narrow circular opening in the window shutter enters a triangular glass prism in the main section position, being therewith transformed into

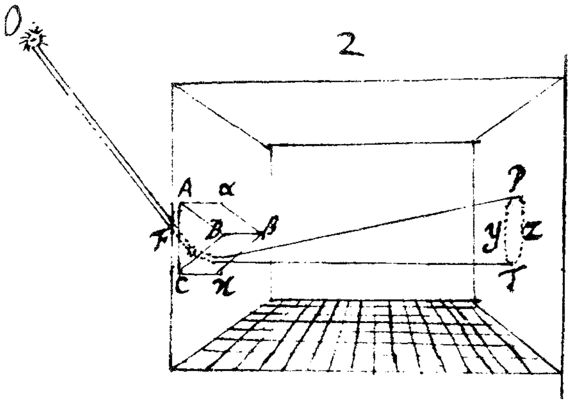


Fig. 1. An early drawing of Newton's fundamental experiment in its most simple form. Reproduced from Ref. 2, p. 32.

a divergent bundle of "spectral rays," which give rise to the image of a "spectrum" on the opposite wall (Fig. 1).

Experimentum V (*The crossed prism experiment*): A second prism is placed behind the first one, the length axes of both prisms being mutually perpendicular. The "spectrum" reappears on the wall, being only displaced and turned a certain angle. This indicates, according to Newton, that the spectral rays are "homogeneous," giving rise to no secondary order dispersion. The greater refrangibility of the blue rays as compared with the red ones explains the tilted position of the spectrum (Fig. 2).

Experiment V may be criticized because of its unnecessary complexity. The cross refraction occurs because the planes of two successive refractions do not coincide, but this may equally well be obtained by means of one single prism only, provided that the incident ray enters obliquely to the length axis of the prism. Newton, being preoccupied by certain advantages of the main section position in prism experiments, overlooked the drawback in this particular case. If he had come upon the simpler procedure, he should have been more likely to discover certain general properties of color transitions due to dispersion. This might not have been without consequences for the further development of the optical science.

Experimentum VI (*The Experimentum Crucis*): Newton was occupied by this experiment throughout all his creative period, and he suggested various versions of it, one of them foreshadowing the modern spectroscope.[5] We shall consider the version suggested in *Opticks:* Parallel, white light enters a system of two successive and mutually perpendicular prisms, between which have been introduced two successive circular slits in fixed positions. From the divergent bundle of spectral rays spreading out from the first slit, one particular ray passes through the exit slit/prism system. By turning the first prism about its length axis, the various spectral rays will pass through the exit system in due order, and since slit positions are fixed, the specific refrangibilities of spectral rays can be easily compared (Fig. 3).

These experiments bring to mind the fact that Newton's investigations were directed towards the behavior of "Rays." The "Ray" represented light *qua* physical object. This was explicitely stated already in the first Definition:

> . . . The least Light or Part of Light, which may be stopp'd alone without the Rest of the Light, or propagated alone, or do or suffer anything which the Rest of the Light doth not or suffers not, I call a Ray of Light.

With this definition Newton took an important step away from the old established geometrical image optics into physical optics. At the same time he introduced the above mentioned terminological confusion, namely by applying the operational rules of the old image optics upon a reinterpreted ray concept. Our textbooks still suffer from that confusion.

The opticians of the Euclidean school, such as Kepler, were not concerned with the physical nature of light nor of vision. They dealt with certain general, geometrical properties of seen images. The actual, perceptive attributes of seen images, such as color, were usually not drawn in. The images of ancient geometrical optics should

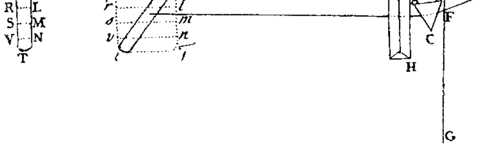


Fig. 2. The crossed prism experiment. Reproduced from *Opticks*.

be interpreted as *potential images*, i.e., images which appear when certain appropriate "boundary conditions" for image seeing are satisfied. It was not the task of geometrical optics to explicate those boundary conditions, but the implicit knowledge about their existence in general was contained in the axiomatic statements which the deductive system rested upon.

The *ray* was introduced as a purely geometrical operator, the behavior of which was completely determined by a set of axioms and definitions dealing with certain geometrical laws governing image seeing. The ray can be interpreted as a potential direction of sight, and this implicit definition explains the experimental origin of the basic knowledge laid down in the axioms. The ray may also be interpreted as a potential direction of image projection, and in the following we shall make use of this in connection with the introduction of slide projectors. Admittedly, we then enter the realm of that highly refined image concept established by Kepler, according to which the image arises in the crossing point of rays, but in the following we shall avoid it and move within a simplified Euclidean ray/image terminology. Expressions like, for instance, "a red ray" means nothing more there than that a red spot can be seen or projected along that particular direction. Among the various properties of the ray operator we shall be concerned with an inherent symmetry which was laid down in the axiom III of Euclid's *Catoptrics*[6]:

> If an image is seen in the plane mirror, the heights of eye and object above the mirror are in the same ratio as the parts of the ray between them. [Figure 4.]

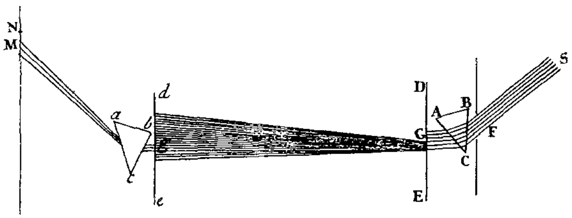


FIG. 3. *Experimentum Crucis*. Reproduced from *Opticks*. (The dispersed light leaving the prism surface AC is erroneously drawn as parallel light.)

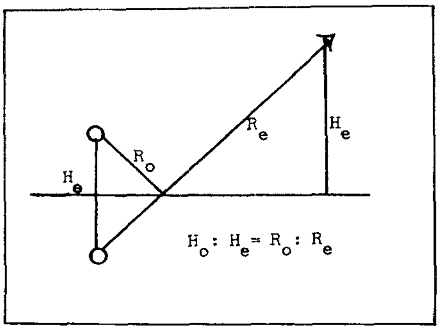


FIG. 4. Euclid's axiom III concerning mirror images.

The axiomatic character of this axiom is revealed when the. following two empirical implications are taken into account: uniqueness of the ray, as well as its invariance against interchange of eye and object. Such implications derive from the experimental experience of the fact that two eyes looking at each other are looking along the same ray. In accordance with the deductive structure of Euclidean optics, the law of equal angles of incidence and reflection is derived from axiom III by purely geometrical reasoning, whereas we are accustomed to introduce this rule as a fundamental natural law concerning the behavior of rays.

No traces have been left of a Euclidean treatment of the phenomena of dispersion. Whether this is due to the lack of appropriate ideas or to the loss of appropriate manuscripts shall not be discussed here. Seemingly, at least, it was Newton who, together with his more or less forgotten contemporaries, succeeded in incorporating the dispersion phenomena into the optical science. The decisive step was taken with the introduction of the concept of "refrangibility." The nature of refrangibility was then further explicated by the introduction of two axiomatic natural laws, that of reversibility of the ray and that of the constant sine ratios:

> If the refracted beam be returned directly back to the Point of Incidence, it shall be reflected into the Line before described by the incident Ray. [Axiom III.]
>
> The Sine of Incidence is either accurately or very nearly in a given Ratio to the Sine of Refraction. [Axiom V.]

Newton's axiom of reversibility is dictated by his belief in the ray as the appropriate object of physical inquiry. This idea led him to the idea that

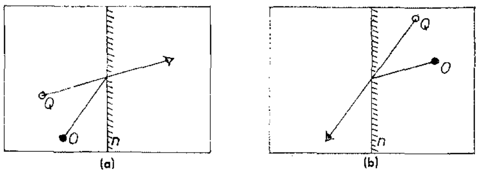


FIG. 5. A modified axiom of reversibility. An object O (*in casu* a boundary or a point) viewed across a plane optical interface n appears as the image Q, (*in casu* a transition of images).

white light consists of a pre-established ensemble of colored rays, which on refraction appear neatly ordered, according to an invariant, universal principle of harmony. From the invariance of this order he concluded that the achromatic lens telescope is impossible. Newton did not distinguish between appropriate classes of boundary conditions for image appearance. In his thought, the image was a secondary effect. He did not realize that in any optical instrument the actual object of our inquiry is the image.

Since it was Euclidean optics which implicitly recognized the existence of boundary conditions for image appearance, the question arises if the Euclidean ray concept may be extended to include the phenomena of dispersion. A comparison with the case of mirror images offers a clue. We have learned that the properties of the reflecting surface determine a complete set of images which can be dealt with systematically by means of the ray operator provided that the appropriate rules of operation have been established. By analogy, therefore, we introduce the "refracting surface" which again determine a complete set of images. Taking into account the general properties of the ray operator, uniqueness and invariance against interchange of eye and object, we suggest an axiom of reversibility which applies to directions of sight quite generally:

> If a particular object (O), seen across an optical interface (n) appears as a particular image (Q), then, after interchange of eye and object, the same image (Q) appears in the direction determined by the ray (Fig. 5).

This axiom of reversibility should be accomplished with the semiempiric law of constant sine ratios (applied in this case to the refracted direction of sight of the particular image Q) before we are ready to go to the experiment itself, which alone can decide the correctness of our concepts.—What actually happens, however, when objects are viewed across an optical interface, is rather disturbing: Apart from being displaced, the boundaries of the objects are transformed into continuous color transitions.[7] Does this mean that the very image concept itself breaks down? Not necessarily; insofar as the color transitions consist of distinguishable steps or "cuts," each particular cut may be considered as one particular image of the particular object boundary considered. The axiom of reversibility together with the law of constant sine ratios then applies to each particular color cut (considered as one particular image). After this, dispersion can be defined as a property of the optical interface, according to which each particular potential direction of sight on one side of the interface corresponds with an angular set of potential directions of sight on the other side. Of course, what is *actually* seen along the particular potential directions of sight is determined not by the properties of the interface, but by the properties of the particular object being introduced—an aperture, for instance—and by the state of the eye.

The latter principle leads to the important ex-

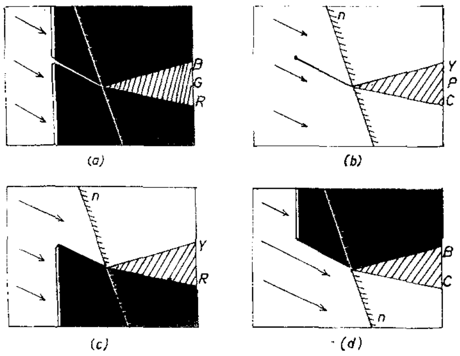


FIG. 6 (a)–(d). Four cases of dispersion by various types of apertures, but one and the same optical interface. Parallel white light enters from the left side. Figures (a) and (b) show the dispersion of beams of light and shadow, respectively. Figures (c) and (d) show the dispersion of asymmetrically delimited "rays." Geometrically all cases are identical, only the color names change. In this and the following figures, the following color terminology is used: B, G, and R refer to spectral blue, green, and red, respectively. Y, P, and C refer to unsaturated yellow, purple, and cyan, respectively. Bl and Wh refer to black and white, respectively.

periments with "boundary colors," the "inverted spectrum," and related phenomena. These experiments have not yet been fully explored, and the reason for the disinterest may be sought in the circumstance that so far, nobody has related them to a more general ray concept.[8]

The inverted spectrum occurs as the dispersed image of a black spot in a bright surround, i.e., when a narrow beam of shadow passes across the interface. The boundary colors arise from the dispersion of asymmetrically delimited images, for instance boundary lines between light and shadow. The inverted spectrum is systematically related with the ordinary one in that the colors of the two spectra are mutually and pairwise complementary. The boundary colors also occur in mutually complementary pairs, according to the asymmetrical boundary conditions. In these experiments, therefore, rays which are complementarily colored have the same refrangibility [Figs. 6(a)-(d)].

These phenomena can be shown in a lecture hall by means of a slide projector with a triangular flint glass prism placed before the objective lens. The images to be dispersed are then introduced as slides. The main principles appear from simple black and white slides showing bright spots in a dark surround and vice versa or other ones showing straight boundary lines between bright and dark areas. Analogous slides in color are recommended as a further demonstration of the consistency and lawfulness of this class of dispersion phenomena.

In the light of the general properties of the dispersion phenomena, we shall consider Newton's abovementioned experiments once more.

The crossed prism experiment: This experiment was suggested by Newton as a test of the postulated homogeneity of spectral rays. However, the question then arises if the test applies to directions of sight in general, i.e., to a more general ray concept. Here, as elsewhere, the last word belongs to the experiment, and in fact, if a system of two mutually perpendicular prisms is mounted before the objective of the slide projector, it is easily verified that the inverted spectrum behaves in exactly the same way as the ordinary one. They are both displaced and turned to the same degree without the occurrence of secondary dispersion of the particular spectral rays. The yellow rays of the inverted spectrum behave exactly as their blue complementaries of the ordinary one, and the same holds true for all pairs of complementary colors. It appears further, that all kinds of "spectra" which arise from the images of arbitrarily colored spots in arbitrarily colored surrounds demonstrate the same geometrical lawfulness. In this type of experiments no systematic correlation between color and refrangibility can be recognized. The angular extension of the dispersion is determined by the refractive properties of the prism system, while the color distribution is determined by the type of "aperture" being introduced *qua* slide image.

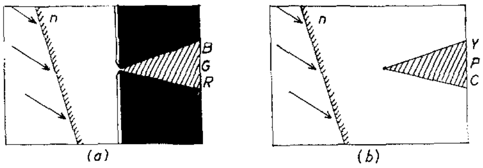


FIG. 7 (a)-(b). Parallel white light passes across a plane optical interface, being therewith transformed into dispersed light. The ordinary and inverted apertures being introduced into the dispersed light give rise to the ordinary and inverted spectra, respectively.

The parallel prism experiment (*Experimentum Crucis*): This experiment was suggested as a test of the theorem of the specific refrangibilities of the spectral rays. More than once it has been referred to as the most heavy weighing argument in favour of Newton's theses.[9] However, again it has been overlooked that the experiment demonstrates a general property of the Euclidean directions of sight. A closer analysis of the experiment reveals a consistent and symmetrical structure of the class of dispersion phenomena, so we shall spend the necessary time to develop the main principles step by step.

As a beginning, we have to agree on some boundary conditions and terms. We assume that parallel white light is passing across an optical interface. This implies, as we have learned, that a unique potential direction of sight is transformed into an angular set of potential directions of sight. The color distribution shall be controlled by means of certain apertures. We may be looking against those apertures, or the corresponding images may be projected on a white screen. (In these experiments we are not looking across the interface.)

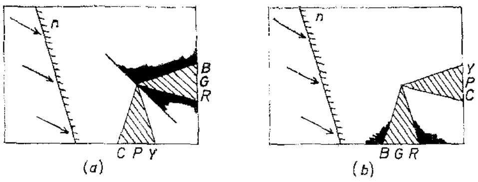


FIG. 8 (a)–(b). Reflecting surfaces have been introduced. Figure (a) shows the effect of a reflecting surface with a narrow, transmitting opening. Figure (b) shows the effect of a small reflecting obstacle making the same angle with the incident light. It appears that in both cases mutually complementarily colored rays are symmetrically spaced about the reflecting surface.

In order to simplify the description we shall deal with only two main types of circular apertures, the "ordinary" and the "inverted" ones. Obviously, the ordinary and inverted apertures give rise to ordinary and inverted spectra, respectively [Fig. 7(a)–(b)].

On the other hand, the effects of these apertures may be produced by means of reflecting surfaces as well. A symmetrical relation is then revealed: A reflecting obstacle reflects an ordinary spectrum, while the shadow appears as an inverted spectrum. A narrow opening in an extended reflecting surface transmits an ordinary spectrum, "reflecting" an inverted one. In both cases, mutually complementarily colored rays are symmetrically spaced about the reflecting surface [Figs. 8(a)–(b)].

We then have to consider various combinations of apertures:

Two successive ordinary apertures reproduce the ordinary spectrum in a highly reduced form, namely as a single spectral line. In spectroscopical language, the respective particular spectral line has been "isolated." Conversely, two successive inverted apertures cause a superposition of two inverted spectra, with only one particular inverted spectral line not being disturbed [Figs. 9(a)–(b)].

Combinations of opposite apertures: An ordinary aperture transforms an inverted spectrum into an ordinary one with a dark quasiabsorption line, the position of the dark line within the spectrum being determined by the mutual positions of the apertures. In spectroscopical language, the respective inverted spectral ray has been "resolved." The dark line is explained by the mutual complementarity between corresponding angular positions within the inverted and the ordinary spectra, respectively. According to the principle of complementarity, each position within the inverted spectrum is characterized by the absence of the corresponding wavelength of ordinary spectrum. On the other hand, in a Euclidean language the transformation is also plausible: The combination of opposite apertures causes a heavy disturbance of the boundary conditions for image appearance; a bright surround is changed into a dark one, or vice versa. Accordingly, we suggest an analogous transformation of an ordinary spectrum into an inverted one: If an ordinary spectral ray from a reflecting obstacle is passed from behind through the opening of a reflecting surface, it will add to its own complementary in the inverted spectrum being reflected from the front side. The net result is an inverted spectrum with a white "reflection line" [Fig. 10(a)–(b)].

Now, at last, we recognize the counterpart of the *Experimentum Crucis* itself, in which we are able to demonstrate the specific degrees of refrangibilities of the rays of the inverted spectrum. According to the boundary conditions, the rays of the inverted spectrum appear in a bright surround only. To maintain this condition is a matter of the appropriate choice of apertures. In Fig. 11 we have shown that two successive reflecting apertures give rise to the desired result: one particular ray of the inverted spectrum within its proper surround. This ray will therefore pass across renewed optical interfaces without any sign of dispersion. Further, it always has the same degree of refrangibility as its complementary counterpart of the ordinary spectrum.

A final remark. The terminological ambiguity, introduced by Newton and still dominating our teaching, derives from inadequate ideas on the

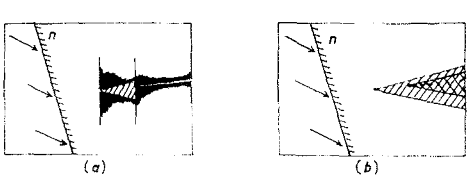
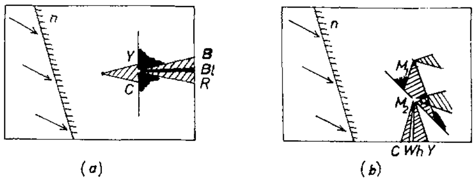


FIG. 9 (a)–(b). Combinations of two successive ordinary and inverted apertures, respectively. In the former case a single spectral ray is isolated. In the latter case two inverted spectra are being superposed.

nature of seen images. Newton considered the image to be a secondary effect, the laws of which are governed by the properties of a closed set of "primary rays." It never really struck him that these rays appear only under certrain highly specified conditions of observation. It is a remarkable fact that Newton's great predecessors, such as Euclid, Descartes, and even Kepler, did not share Newton's view on the relation between rays and images. In Cartesian optical terminology, for instance, the refractive index was considered as a property of the optical interface.[10] The actual displacement of a seen image was taken as a measure of the magnitude of the refractive index.

The reason for Newton's success must be found, after all, in the circumstance that his instincts were wiser than his words. The real axiom underlying Newton's *Opticks* is this: The color seen by the eye can be considered as a physically significant observation insofar as the conditions of observation are kept constant. Under such conditions the eye appears as a reliable instrument of measurement. In principle, Newton might have arrived at the same physical models, even if he had worked systematically with inverted apertures, i.e., under "bright room" conditions, but in that case he should have come upon another color terminology.

The basic contradiction involved in Newtonian terminology seems to be this: Newton thought that he explained the existence of a spectrum by means of a physical model of the light, whereas he in fact used the image of the spectrum to explain one possible physical model of the light.



(a) (b)

FIG. 10 (a)-(b). Combinations of opposite apertures. Figure (a) shows that an ordinary aperture transforms one particular inverted spectral ray into an ordinary spectrum with a quasiabsorption line. In Fig. (b) an ordinary spectral ray (from a reflecting obstacle) is transformed into an inverted spectrum with a white "reflexion line," namely by being superimposed upon an inverted spectrum (from a reflecting surface).

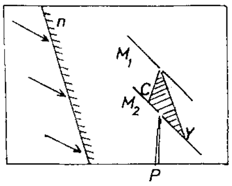


FIG. 11. The counterpart of *Experimentum Crucis*. The reflecting aperture M_1 produces an inverted spectrum YC, from which one particular ray (P) passes through the narrow opening in M_2, being therewith transformed into an ordinary spectrum with a quasiabsorption line. The latter spectrum is added to the inverted spectrum arising from the frontside of the reflecting surface M_2. The net result of this superposition is the ray P of an inverted spectrum in a white surround.

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[1] I. Newton, *Opticks*, (1704) (Dover Edition, New York, 1952).

[2] J. A. Lohne and B. Sticker, *Newtons Theorie der Prismenfarben* (Werner Fritsch, Munich, 1969). This work contains numerous historical as well as bibliographical references.

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[4] Reference 3, Vol. II, p. 257.

[5] J. A. Lohne, Notes Rec. Roy. Soc. (London) **23**, 169 (1968).

[6] Euclid, *L'Optique et la catoptrique*, edited by P. V. Eecke (Albert Blanchard, Paris, 1959).

[7] The colored bands or fringes occurring in imperfect optical systems or when dividing lines between lighter and darker areas are viewed through a prism, belong to the class of "boundary colors." For a detailed discussion of the respective spectral distributions from the standpoint of colorimetry see P. J. Bouma, *Physical Aspects of Colour* (Phillips Technical Library, Eindhoven, Netherlands, 1947), p. 126 ff. From the standpoint of color vision the "boundary colors" are closely related to the class of "optimal colors," the theory of which was first developed by Schroedinger [Ann. Physik **62**, 603 (1920)].

[8] The socalled "inverted Spectrum" goes back to the investigations of Goethe [*Beiträge zur Optik* (1791), *Entwurf einer Farbenlehre* (1810)]. For more recent studies see, for instance, A. Kirschmann [Physik. Z. **18**, 195 (1917)] and K. Miescher and R. Rometsch [Experientia **6**, 301 (1950)].

[9] H. Helmholtz, *Physiologische Optik* (Leopold Voss, Leipzig, 1867), p. 268.

[10] J. A. Lohne and B. Sticker, Ref. 2, p. 35.